ENUMATH 2023

LISBOA PORTUGAL

ϕ-FEM : A fictitious domain method for FEM on domains defined by level-sets

M. Duprez[1], V. Lleras[2], K. Vuillemot[1,2] and A. Lozinski[3],

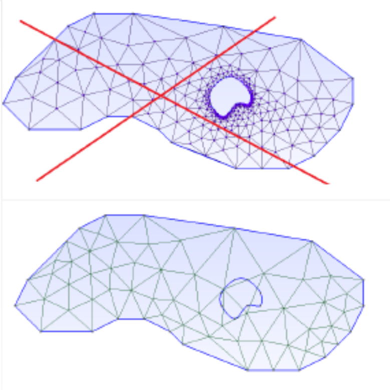
[1]Inria, Strasbourg, France
[2]IMAG, Montpellier, France
[3]Laboratoire de Mathématiques de Besançon, France

Inria INVENTEURS DU MONDE NUMÉRIQUE · UNIVERSITÉ DE MONTPELLIER · UBFC UNIVERSITÉ BOURGOGNE FRANCHE-COMTÉ · PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

Outline

1. **Motivation and previous works**
2. **ϕ-FEM method for elasticity problems**
   1. **With Dirichlet conditions**
   2. **With Neumann conditions**
   3. **With mixed conditions**
3. **Particulate flows**
4. **Summary and outlook**

Finite elements on non-matching grids : motivations



Possible uses of non-matching grids

- A simpler treatment of complex geometries, cracks, material interfaces, ...
- Inverse problems, shape optimization : geometrical features of *a priori* unknown shape
  (domain changing on iterations)
- Fluid-Structure interaction, particulate flows, ...
  (domain changing in time)

Previous works, first works

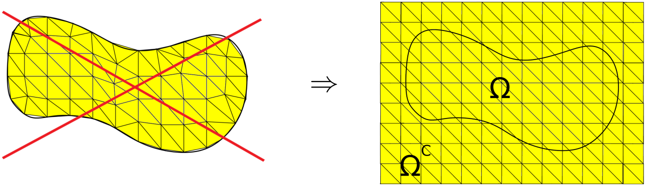
References

Saul'ev '63 (Dirichlet), Astrakhantsev '78 (Neumann), Glowinski et al. 1990's (several extensions and variants)

Formal idea

Fictitious extension of the solution to a surrounding box

Boundary conditions are imposed by Lagrange multipliers or Penalization



☺ **Non-conform mesh (complex and time varying geometry)**

☹ **Large FE matrix** and **bad cond. number**

☹ **The extension is only $H^{1+\varepsilon}$ regular => poor accuracy $O(\sqrt{h})$**

References

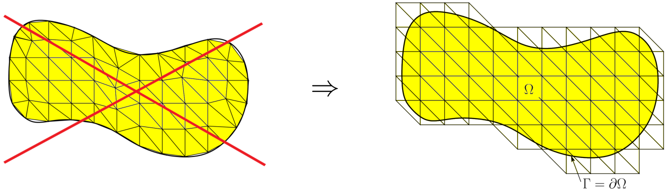
Moes-Bechet-Tourbier 2006, Haslinger-Renard 2009

Formal idea

Cut shape function : $\psi_k \longrightarrow \psi_k \mathbb{1}_\Omega$

Boundary condition on Γ : Lagrange multiplier

Conditioning of the matrix : stabilization on the boundary



☺ **Small FE matrix**

☺ **Good conditioning number**

☹ **Non-classical shape functions** and **discontinuity** in the integrals

Previous works, Cut FEM : partial integration on the cells

References

Burman-Hansbo 2010-2014 (and later works)

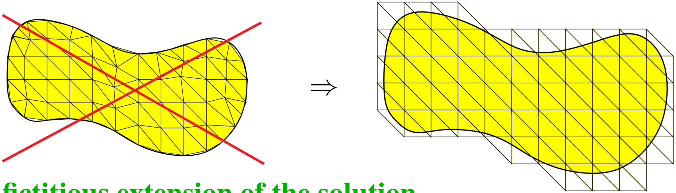
Formal idea

Partial integration on the cell near the boundary

Lagrange multiplier or penalization for the boundary conditions

Conditioning of the matrix : Appropriate stabilization

$\rightarrow$ (Ghost penalty)



☺ **No fictitious extension of the solution**

☺ **Optimal accuracy**

☺ **Standard shape functions**

☹ **Not straigtforward to implement : need to evaluate the integrals on cut mesh elements**.

Formal idea (Dirichlet)

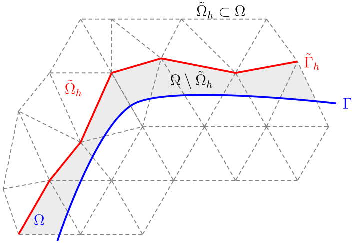
Taylor expansion of the bound. cond.

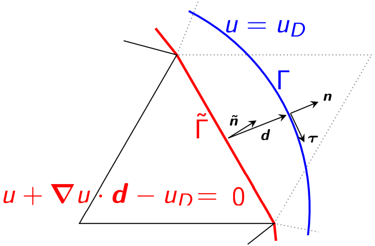
Reference

Main-Scovazzi 2017

Real boundary condition on Γ : $\boldsymbol{u = u_D}$

Discrete bound. cond. on $\widetilde{\Gamma}$: $\boldsymbol{u + \nabla u \cdot d = u_D}$





☺ **No cut integral**

☺ **Optimal accuracy in the Dirichlet case**

☹ **Theory : the discr. bound.y has to be closed to the real bound.**

☹ **Theory : no theory for Neumann**

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Goal of ϕ-FEM :

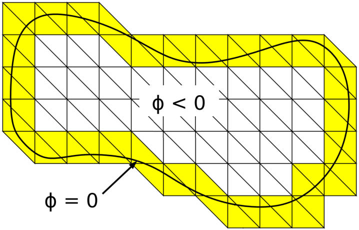
- Non matching grid
- Optimal acuracy
- Straigtforward to implement

What is the idea of ϕ-FEM ?

Hypothesis :

Assume that Ω and Γ are given by a **level-set** function ϕ :

$$\Omega := \{\phi < 0\} \text{ and } \Gamma := \{\phi = 0\}.$$



Good candidat : the signed distance

Idea of ϕ-FEM :

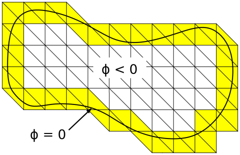
Include the Level-set function in the formulation
to take into account the boundary conditions

The domain Ω and its boundary Γ be given by a **level-set** function ϕ :

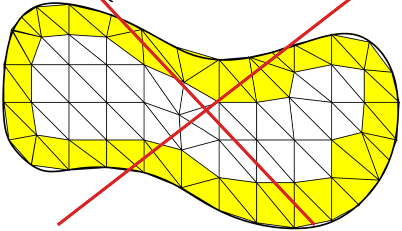
$$\Omega := \{\phi < 0\}$$

and

$$\Gamma := \{\phi = 0\}$$

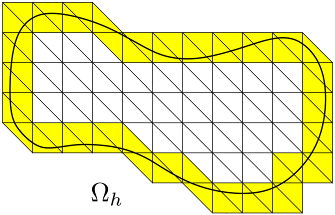


$$\exists\, u \text{ s.t. } \begin{cases} -\operatorname{div}(\sigma(u)) = f \text{ on } \Omega \\ \boldsymbol{u = 0 \text{ on } \Gamma} \end{cases}$$



$\exists\, v$ s.t. $-\operatorname{div}(\sigma(\phi v)) = f$ on $\boldsymbol{\Omega_h}$

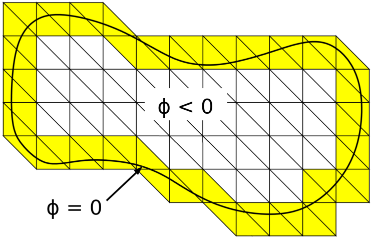
Then $\boldsymbol{u = \phi v}$



Reminder :

$\Omega := \{\phi < 0\}$ and $\Gamma := \{\phi = 0\}$

$\Omega_h \supset \Omega$ is slightly larger than Ω



The ϕ-FEM ansatz :

$$u = \phi v; \quad u \text{ and } v \text{ are extended to } \Omega_h$$

Strong formulation :

$$-\operatorname{div}(\sigma(\phi v)) = f \text{ on } \Omega_h$$

The weak formulation for $v \in H^1(\Omega_h)$ with f given on Ω_h :

$$\int_{\Omega_h} \sigma(\phi v) \cdot \nabla(\phi w) - \int_{\partial\Omega_h} \frac{\partial}{\partial n}(\phi v)\phi w = \int_{\Omega_h} f\phi w, \qquad \forall w \in H^1(\Omega_h)$$

This problem is ill posed. Nevertheless, we use it as a starting point for the FE discretization and stabilize...

Dirichlet BC : ϕ-FEM on the fully discrete level

Set $u_h = v_h \phi_h$ where $v_h \in V_h$ (the conforming P_k FE space on $\mathcal{T}_h$) solves

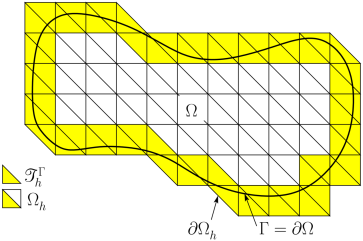
$$\int_{\Omega_h} \sigma(\phi_h v_h) \cdot \nabla(\phi_h w_h) - \int_{\partial\Omega_h} \frac{\partial}{\partial n}(\phi_h v_h)\phi_h v_h + G_h(v_h, w_h) = \int_{\Omega_h} f \phi_h w_h + G_h^{rhs}(w_h), \quad \forall w_h \in V_h$$

with $\phi_h = I_h(\phi)$ (interpolation to FE of degree $\geq k$).
G_h and G_h^{rhs} stand for the the **ghost penalty**

$$G_h(v_h, w_h) = \sigma h \sum_{E \in \mathcal{F}_\Gamma} \int_E \left[\frac{\partial}{\partial n}(\phi_h v_h)\right] \left[\frac{\partial}{\partial n}(\phi_h w_h)\right]$$

$$+\sigma h^2 \sum_{T \in \mathcal{T}_h^\Gamma} \int_T \operatorname{div}(\sigma(\phi_h v_h)) \operatorname{div}(\sigma(\phi_h w_h))$$

$$G_h^{rhs}(w_h) = -\sigma h^2 \sum_{T \in \mathcal{T}_h^\Gamma} \int_T f \operatorname{div}(\sigma(\phi_h w_h))$$



Notations :

$\mathcal{T}_h^\Gamma = \{T \in \mathcal{T}_h : T \cap \Gamma_h \neq \varnothing\}$ $\quad (\Gamma_h = \{\phi_h = 0\})$;
$\mathcal{F}_\Gamma = \{E$ (an internal edge of $\mathcal{T}_h$) such that $\exists T \in \mathcal{T}_h : T \cap \Gamma_h \neq \varnothing$ and $E \in \partial T\}$.

Dirichlet BC : An *a priori* error estimates

Theorem (Duprez-Lozinski'20)

Let $u \in H^{k+2}(\Omega)$ be the exact solution and $u_h := \phi_h v_h$ be given by ϕ-FEM. Under the assumptions above (and some more technical ones), taking $\sigma > 0$ big enough,

$$|u - u_h|_{1,\Omega} \le Ch^k \|f\|_{k,\Omega_h}, \qquad \|u - u_h\|_{0,\Omega} \le Ch^{k+1/2} \|f\|_{k,\Omega_h}$$

with $C = C(\text{regularity of } \mathcal{T}_h, \text{regularity of } \phi)$.

Remark :

1. Numerical experiments show in fact $\|u - u_h\|_{0,\Omega} \le Ch^{k+1}$
2. For non-homogeneous Dirichlet conditions u_D, we replace ϕv by $\phi v + u_D$
3. Optimal conditioning number

Lemma (Key tool : Hardy inequality)

We assume that the domain Ω is given by the level-set ϕ regular enough.
For any $u \in H^{k+1}(\mathcal{O})$ vanishing on Γ :

$$|u/\phi|_{k,\mathcal{O}} \le C\|u\|_{k+1,\mathcal{O}}.$$

Dirichlet BC penalized version

Ansatz : $\boldsymbol{u}_h = \frac{1}{h}\phi_h \boldsymbol{w}_h$ on Ω_h^Γ

Duality : it is impose in a least square manner.

Find $\boldsymbol{u}_h \in V_h$ ($\mathbb{P}^k$ FE space on the discrete domain Ω_h)
and $\boldsymbol{w}_h \in Q_{h,D}$ ($\mathbb{P}^k$ FE space on the cells of the boundary Ω_h^Γ)

$$\int_{\Omega_h} \boldsymbol{\sigma}(\boldsymbol{u}_h) : \nabla \boldsymbol{v}_h - \int_{\partial\Omega_h} \boldsymbol{\sigma}(\boldsymbol{u}_h)\boldsymbol{n} \cdot \boldsymbol{v}_h + \frac{\gamma}{h^2} \int_{\Omega_h^\Gamma} (\boldsymbol{u}_h - \frac{1}{h}\phi_h \boldsymbol{w}_h) \cdot (\boldsymbol{v}_h - \frac{1}{h}\phi_h \boldsymbol{z}_h)$$

$$+ J_h^{lhs}(\boldsymbol{u}_h, \boldsymbol{v}_h) = \int_{\Omega_h} \boldsymbol{f} \cdot \boldsymbol{v}_h + J_h^{rhs}(\boldsymbol{v}_h), \quad \forall \boldsymbol{v}_h \text{ on } V_h, \boldsymbol{z}_h \text{ on } Q_{h,D}$$

with the same stabilization terms

$$J_h^{lhs}(\boldsymbol{u}, \boldsymbol{v}) := \sigma h \sum_{E \in \mathcal{F}_h^\Gamma} \int_E [\boldsymbol{\sigma}(u) \cdot \boldsymbol{n}] \cdot [\boldsymbol{\sigma}(v) \cdot \boldsymbol{n}] + \sigma h^2 \sum_{T \in \mathcal{T}_h^\Gamma} \int_T (div \boldsymbol{\sigma}(\boldsymbol{u})) \cdot (div \boldsymbol{\sigma}(\boldsymbol{v}))$$

$$J_h^{rhs}(\boldsymbol{v}) := -\sigma h^2 \sum_{T \in \mathcal{T}_h^\Gamma} \int_T f \cdot (div \boldsymbol{\sigma}(\boldsymbol{v}))$$

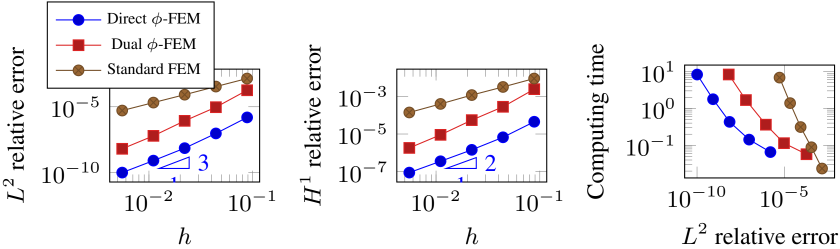
Dirichlet BC penalized version : numerical simulations

$\mathcal{O} = [0,1] \times [0,1]$, $\phi(x,y) = \frac{-1}{8} + (x-0.5)^2 + (y-0.5)^2$.

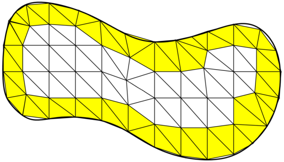
Parameters : $E = 2$ and $\nu = 0.3$, and $\gamma = \sigma_D = 20.0$, $k = 2$.

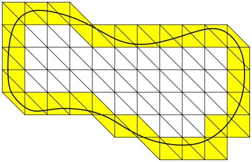
exact solution : $\boldsymbol{u} = \boldsymbol{u}_{ex} := (\sin(x)\exp(y), \sin(y)\exp(x))$

We extend $\boldsymbol{u}^g$ from Γ to Ω_h (direct method) or Ω_h^Γ (dual method) : $\boldsymbol{u}^g = \boldsymbol{u}_{ex}(1+\phi)$



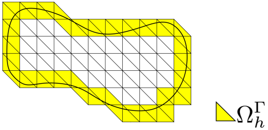
Remark : We have a **better convergence than standard FEM**





Neumann BC : ϕ-FEM scheme

Since $n \sim \nabla\phi/|\nabla\phi|$, the Neumann boundary problem can be reformulated as



$$-\operatorname{div}\boldsymbol{\sigma}(\boldsymbol{u}) = f \text{ in } \Omega_h\,,$$
$$\boldsymbol{y} + \boldsymbol{\sigma}(\boldsymbol{u}) = 0 \text{ in } \Omega_h^\Gamma\,,$$
$$\boldsymbol{\sigma}(\boldsymbol{u})\cdot n = 0 \text{ on } \Gamma \Leftrightarrow -\boldsymbol{y}\cdot\nabla\phi + \boldsymbol{p}\phi = 0 \text{ in } \Omega_h^\Gamma\,.$$

The ϕ-FEM scheme is then obtained : Find $(\boldsymbol{u}_h, \boldsymbol{y}_h, \boldsymbol{p}_h) \in V_h \times Z_h \times Q_{h,N}$ such that

$$\int_{\Omega_h} \boldsymbol{\sigma}(\boldsymbol{u}_h) : \boldsymbol{\nabla}(\boldsymbol{v}_h) + \int_{\Gamma_h} (\boldsymbol{y}_h\cdot\boldsymbol{n})\cdot\boldsymbol{v}_h + \gamma_{div}\int_{\Omega_h^\Gamma} \operatorname{div}\boldsymbol{y}_h\cdot\operatorname{div}\boldsymbol{z}_h$$
$$+\gamma_u\int_{\Omega_h^\Gamma} (\boldsymbol{y}_h + \boldsymbol{\sigma}(\boldsymbol{u}_h))(\boldsymbol{z}_h + \boldsymbol{\sigma}(\boldsymbol{v}_h)) + \frac{\gamma_p}{h^2}\int_{\Omega_h^\Gamma}\left(\boldsymbol{y}_h\cdot\nabla\phi_h + \frac{1}{h}\boldsymbol{p}_h\phi_h\right)\left(\boldsymbol{z}_h\cdot\nabla\phi_h + \frac{1}{h}\boldsymbol{q}_h\phi_h\right)$$
$$+\sigma_p h\int_{\Gamma^i}[\boldsymbol{\sigma}(u_h)\cdot\boldsymbol{n}]\,[\boldsymbol{\sigma}(\boldsymbol{v}_h)\cdot\boldsymbol{n}] = \int_{\Omega_h}\boldsymbol{f}\cdot\boldsymbol{v}_h + \gamma_{div}\int_{\Omega_h^\Gamma}\boldsymbol{f}\cdot div\boldsymbol{z}_h$$
$$,\forall\,(\boldsymbol{v}_h, \boldsymbol{z}_h, \boldsymbol{q}_h) \in V_h \times Z_h \times Q_{h,N}.$$

Neumann BC : optimal convergence

Theorem (D.-Lozinski-Lleras 2022)

Suppose that the mesh is quasi-uniform (under some weak assumption), $l \geq k+1$, $\Omega \subset \Omega_h$ and $f \in H^k(\Omega_h)$. Let $u \in H^{k+2}(\Omega)$ be the continuous solution and $(u_h, y_h, p_h) \in W_h^{(k)}$ be the discrete solution. Provided γ_{div}, γ_u, γ_p, σ are sufficiently big, it holds

$$|u - u_h|_{1,\Omega} \leq Ch^k(\|f\|_{k,\Omega_h} + \|g\|_{k+1,\Omega_h^\Gamma})$$

and

$$\|u - u_h\|_{0,\Omega} \leq Ch^{k+1/2}(\|f\|_{k,\Omega_h} + \|g\|_{k+1,\Omega_h^\Gamma})$$

Remark

- non-homo. bound. cond. : $-\boldsymbol{y} \cdot \nabla\phi + \boldsymbol{p}\phi = \boldsymbol{g}|\nabla\phi|$
- The cost of the additional variables $\mathbf{y}$ and p is asymptotically negligible as $h \to 0$ since the band Ω_h^Γ is of width h
- Optimal conditioning number

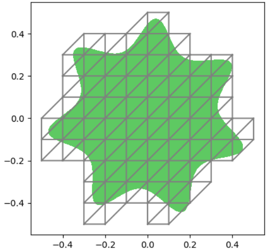
Neumann BC : numerical simulation

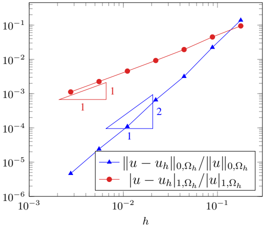
- Real **geometry** : $\Omega := \{\phi < 0\}$
- **Level set** function (polar coordinates) :

$$\phi(r,\theta) = r^4(5+3\sin(7(\theta-\theta_0)+7\pi/36))/2-R^4$$

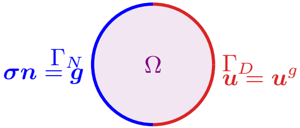
where $R = 0.47$ and $\theta_0 \in [0, 2\pi)$

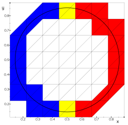
- Surrounding domain : $\mathcal{O} = (-0.5, 0.5)^2$
- **Exact solution** : $u(x,y) := \sin(x)\exp(y)$
- **Source** term : $f := -\Delta u + u$
- Extrapolated **Neumann** boundary condition :
  $\tilde{g} = \frac{\nabla u \cdot \nabla \phi}{|\nabla \phi|} + u\phi$

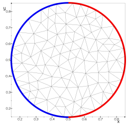




Neumann/Dirichlet (penalized version) BC



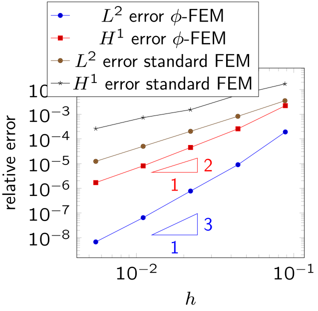


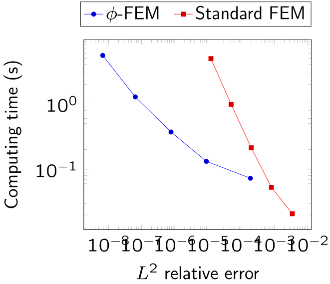


Parameters : $\lambda = 100\ kPa$, $\nu = 0.3$, $\gamma_{div} = \gamma_u = \gamma_p = 1.0$, $\sigma = 0.01$ and $\gamma = \sigma_D = 20.0$.
exact solution : $u_{ex} = (\sin(x) \times \exp(y), \sin(y) \times \exp(x))$
extrapolated boundary conditions

$$\boldsymbol{u}^g = \boldsymbol{u}_{ex} \times (1+\phi), \quad \text{on } \Omega_h^\Gamma \cap \{x \geqslant 0.5\}, \quad \boldsymbol{g} = \boldsymbol{\sigma}(\boldsymbol{u}_{ex}) \times \frac{\nabla\phi}{\|\nabla\phi\|} + \boldsymbol{u}_{ex} \times \phi, \quad \text{on } \Omega_h^\Gamma \cap \{x < 0.5\}$$

where we used $\boldsymbol{u}_{ex} \times \phi$ to add a little perturbation to the exact solution on the boundaries.





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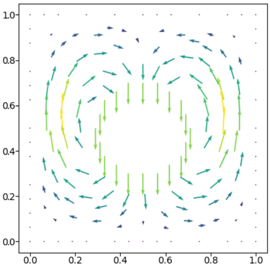
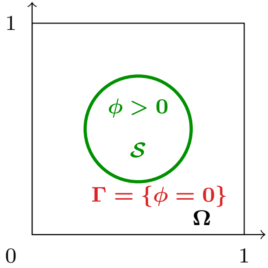
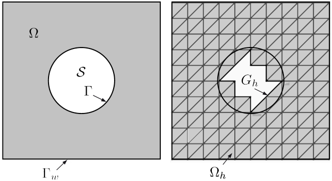
Governing equations :

$$
\begin{aligned}
-2\nu \,\mathrm{div}\, D(u) + \nabla p &= \rho_f g, && \text{in } \Omega\\
\mathrm{div}\, u &= 0, && \text{in } \Omega\\
u &= U + \psi \times r, && \text{on } \Gamma = \{\phi = 0\}\\
u &= 0, && \text{on } \Gamma_w\\
-\int_\Gamma (2D(u) - pI)n + mg &= 0 &&\\
\int_\Gamma (2D(u) - pI)n \times r &= 0 &&\\
\int_\Omega p &= 0 &&
\end{aligned}
$$





Weak formulation :



We define the non-conforming active mesh $\mathcal{T}_h$ on $\Omega_h \supset \Omega$ with its internal boundary G_h.
• u and p can be extended from Ω to Ω_h, an integration by parts gives

$$2\nu \int_{\Omega_h} D(u) : D(v) - \int_{\Omega_h} p \,\mathrm{div}\, v - \int_{\Omega_h} q \,\mathrm{div}\, u - \int_{G_h} (2\nu D(u) - pI)n \cdot v = \int_{\Omega_h} \rho_f g \cdot v.$$

• We make the ansatz $u = \phi w + \chi(U + \psi \times r)$ where ϕ is a level set function for the fluid domain $\Omega = \{\phi < 0\}$, and χ is a sufficiently smooth function on $\mathcal{O}$ such that

$$\begin{cases} \chi = 1 \text{ on the solid } \mathcal{S}, \\ \chi = 0 \text{ on } \Gamma_w. \end{cases}$$

Weak formulation : Setting $u_h = \chi_h(U_h + \psi_h \times r) + \phi_h w_h, \nu = 1$.
Find $u_h \in \mathcal{V}_h^{rbm} = \{\chi_h(V_h + \omega_h \times r) + \phi_h s_h$ with $s_h \in \mathcal{V}_h, V_h \in \mathbb{R}^d, \omega_h \in \mathbb{R}^{d'}\}$ and $p_h \in \mathcal{M}_h = \left\{q_h \in C(\bar\Omega) : q_h|_T \in \mathbb{P}^{k-1}(T) \quad \forall T \in \mathcal{T}_h, \quad \int_\Omega q_h = 0\right\}$ such that

$$c_h(u_h, p_h; v_h, q_h) = L_h(v_h, q_h), \quad \forall v_h \in \mathcal{V}_h^{rbm},\ q_h \in \mathcal{M}_h\,,$$

where the bilinear form c_h is given by

$$\begin{aligned} c_h(u_h, p_h; v_h, q_h) = 2\int_{\Omega_h} D(u_h) : D(v_h) - \int_{\partial\Omega_h} (2D(u_h) - p_h I)n \cdot \phi_h s_h \\ - \int_{\Omega_h} q_h \operatorname{div} u_h - \int_{\Omega_h} p_h \operatorname{div} v_h \\ + \sigma h^2 \sum_{T \in \mathcal{T}_h^\Gamma} \int_T (-\Delta u_h + \nabla p_h) \cdot (-\Delta v_h - \nabla q_h) + \sigma \sum_{T \in \mathcal{T}_h^\Gamma} \int_T (\operatorname{div} u_h)(\operatorname{div} v_h) \\ + \sigma_u h \sum_{E \in \mathcal{F}_h^\Gamma} \int_E [\partial_n u_h] \cdot [\partial_n v_h] + \sigma_u h^3 \sum_{E \in \mathcal{F}_h^\Gamma} \int_E \left[\partial_n^2 u_h\right] \cdot \left[\partial_n^2 v_h\right] \end{aligned}$$

and the linear form L_h is given by

$$\begin{aligned} L_h(v_h, q_h) = \int_{\Omega_h} \rho_f g \cdot \phi_h s_h + \int_{\mathcal{O}} \rho_f g \cdot \chi_h(V_h + \omega_h \times r) + \left(1 - \frac{\rho_f}{\rho_s}\right) mg \cdot V_h \\ + \sigma h^2 \sum_{T \in \mathcal{T}_h^\Gamma} \int_T \rho_f g \cdot (-\Delta v_h - \nabla q_h). \end{aligned}$$

Particulate flow : rate of convergence

Theorem (D.-LLeras-Lozinski 2022)

Let $(u, U, \psi, p) \in H^{k+1}(\Omega)^d \times \mathbb{R}^d \times \mathbb{R}^{d'} \times H^k(\Omega)$ *be the solution to the continuous problem and* $(w_h, U_h, \psi_h, p_h) \in \mathcal{V}_h \times \mathbb{R}^d \times \mathbb{R}^{d'} \times \mathcal{M}_h$ *be the solution to the stabilized scheme. Denoting* $u_h := \chi_h(U_h + \psi_h \times r) + \phi_h w_h$*, the* H^1 *a priori error estimate holds for* $h \leq h_0$

$$|u - u_h|_{1,\Omega\cap\Omega_h} + |p - p_h|_{0,\Omega\cap\Omega_h} \leq Ch^k(\|u\|_{k+1,\Omega} + \|p\|_{k,\Omega})$$

and the error for the translation and the rotation of the solid is the following :

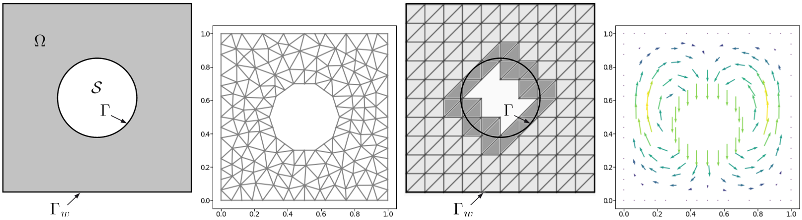
$$|U - U_h| + |\psi - \psi_h| \leq Ch^k(\|u\|_{k+1,\Omega} + \|p\|_{k,\Omega})$$

with some $C > 0$ *and* $h_0 > 0$ *depending on the maximum of the derivatives of* ϕ *and* χ *of order up to* $k + 1$*, on the mesh regularity, and on the polynomial degree* k*, but independent of* h*,* f*, and* u*.*
Moreover, supposing $\Omega \subset \Omega_h$*, the* L^2 *error of the velocity is :*

$$\|u - u_h\|_{0,\Omega} \leq Ch^{k+1/2}(\|u\|_{k+1,\Omega} + \|p\|_{k,\Omega})$$

with a constant $C > 0$ *of the same type as above.*

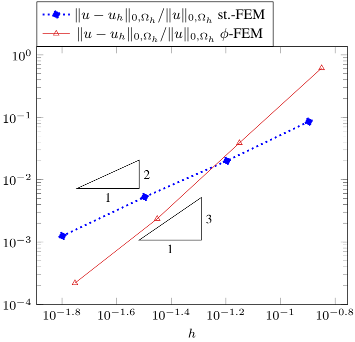
Particulate flow



Parameters : $\phi(x, y) = R^2 - (x - 0.5)^2 - (y - 0.5)^2$, $g = 10$, $\rho_f = 1$, $\rho_s = 2$, $m = \rho_s \pi^2 R^2$, $\sigma = \sigma_u = 20$.

Function χ **:** we consider the radial polynomial of degree 5 on the interval (r_0, r_1) with $r_0 = 0.21$ and $r_1 = 0.45$ such that $\chi(r_0) = 1$ and $\chi'(r_0) = \chi''(r_0) = \chi(r_1) = \chi'(r_1) = \chi''(r_1) = 0$.
Standard FEM : Taylor Hood

Particulate flow



Results

- Optimal convergence
  - ➢ Proof of Poisson-Dirichlet, Poisson-Neumann, Stokes, Heat equation
- Formulation available for any order of approximation
- Well conditioned FEM matrix
- **Simple implementation** in general purpose FEM packages
  - ➢ freeFEM, FEniCS (without plugins)
- **Good ratio speed/error**

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